Normal generators for Mod(S) ☆

S closed, orient. conn. surface of genus $g \geq 2$

$\mathrm{Mod}(S) = \mathrm{Homeo}_+(S) / \text{isotopy}$.

↓

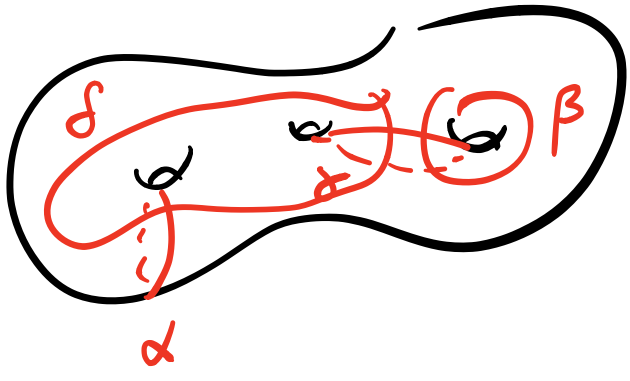
mapping class group.

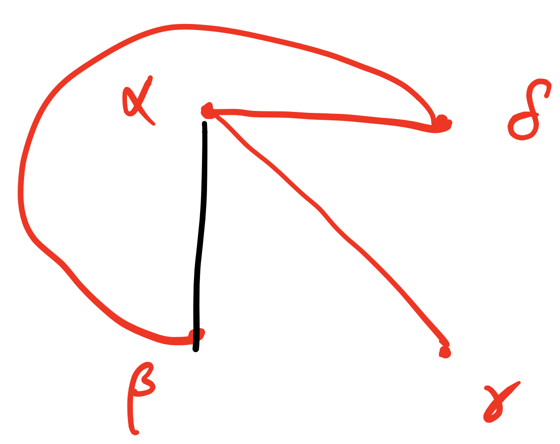
$\mathcal{C}(S)$

curve graph

Vertices = free homotopy classes of ess. s.c.c.'s.

Edges = disjointly represented curves.





$\mathrm{Mod}(S) \curvearrowright \mathcal{C}(S)$ by isometries

Nielsen - Thurston classification

$\mathrm{Mod}(S)$ $\begin{cases} \text{periodic (of fin. order)} \\ \text{reducible} \\ \text{pseudo-Anosov} \end{cases}$

$f \in \mathrm{Mod}(S)$ $\qquad V(\mathcal{C}(S))$

$$\ell_c(f) := \liminf_{n \to \infty} \frac{d_c(\alpha, f^n(\alpha))}{n}$$

Thm (Masur - Minsky)

$$\boxed{f \text{ is p-A} \iff \ell_c(f) > 0.}$$

Thm (Bowditch) $\exists\, m = m(S)$ s/t

$\forall f \in \mathrm{Mod}(S)$, $\ell_c(f) \in \mathbb{Q}$ w/ denom $\leq m$

__Note__ $\ell_c(f^k) = k \cdot \ell_c(f)$

$H \leq \mathrm{Mod}(S)$.
subset

$$L_c(H) := \min\{\ell_c(f) \mid f \in H \ \& \ \underline{f \text{ is p-A}}\}.$$

__Q.__ Can $\underline{L_c(H)}$ distinguish proper normal subgps of $\mathrm{Mod}(S)$ from $\mathrm{Mod}(S)$?

__Th__ (Lanier - Margalit). $\forall g \geq 3$,

$\forall f \in \mathrm{Mod}(S_g)$, $\lambda_f \leq \sqrt{2}$
p-A

$$\Longrightarrow \quad \langle\langle f \rangle\rangle = \mathrm{Mod}(S_g).$$

Th (Gadre-Tsai , 2011)

$$L_c(\mathrm{Mod}(S_g)) \asymp \frac{1}{g^2}$$

Th (Kim-Shin) $L_c(B_n) \asymp \frac{1}{n^2}$

braid group

Th. (B. - Shin)

- $L_c(\mathcal{I}_g) \asymp \frac{1}{g}$

Mod($\odot$)

- $L_c(PB_n) \asymp \frac{1}{n}$.

- Thurston norm

M closed 3-mfld.

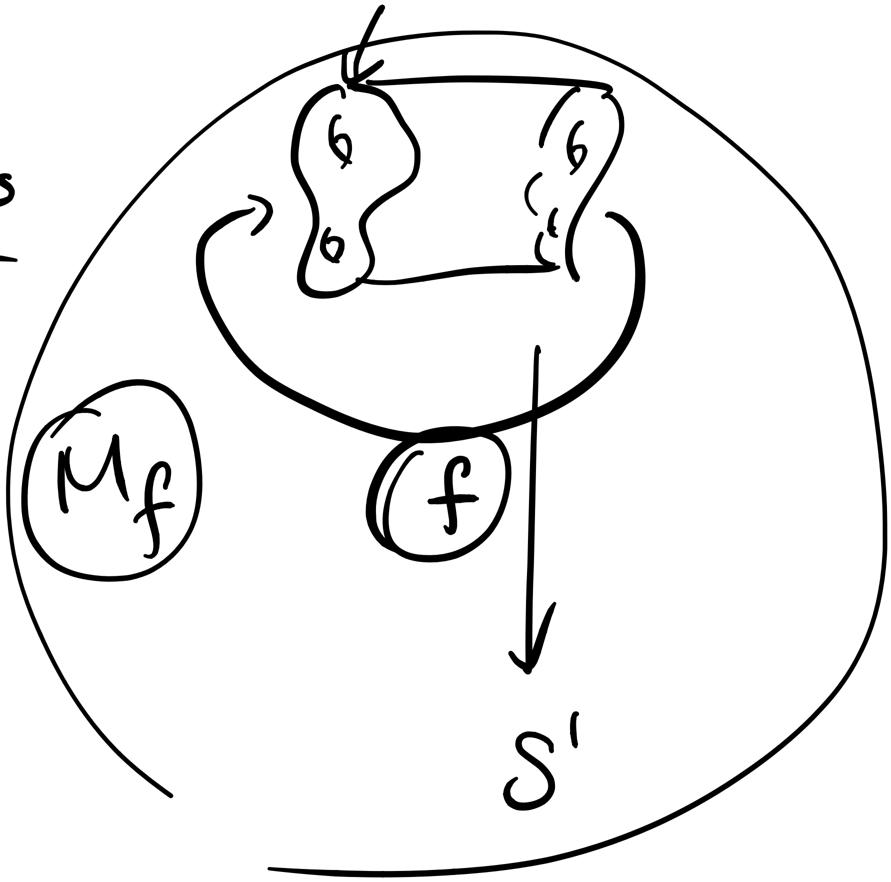
$[S] \in H_2(M;\mathbb{Z})$

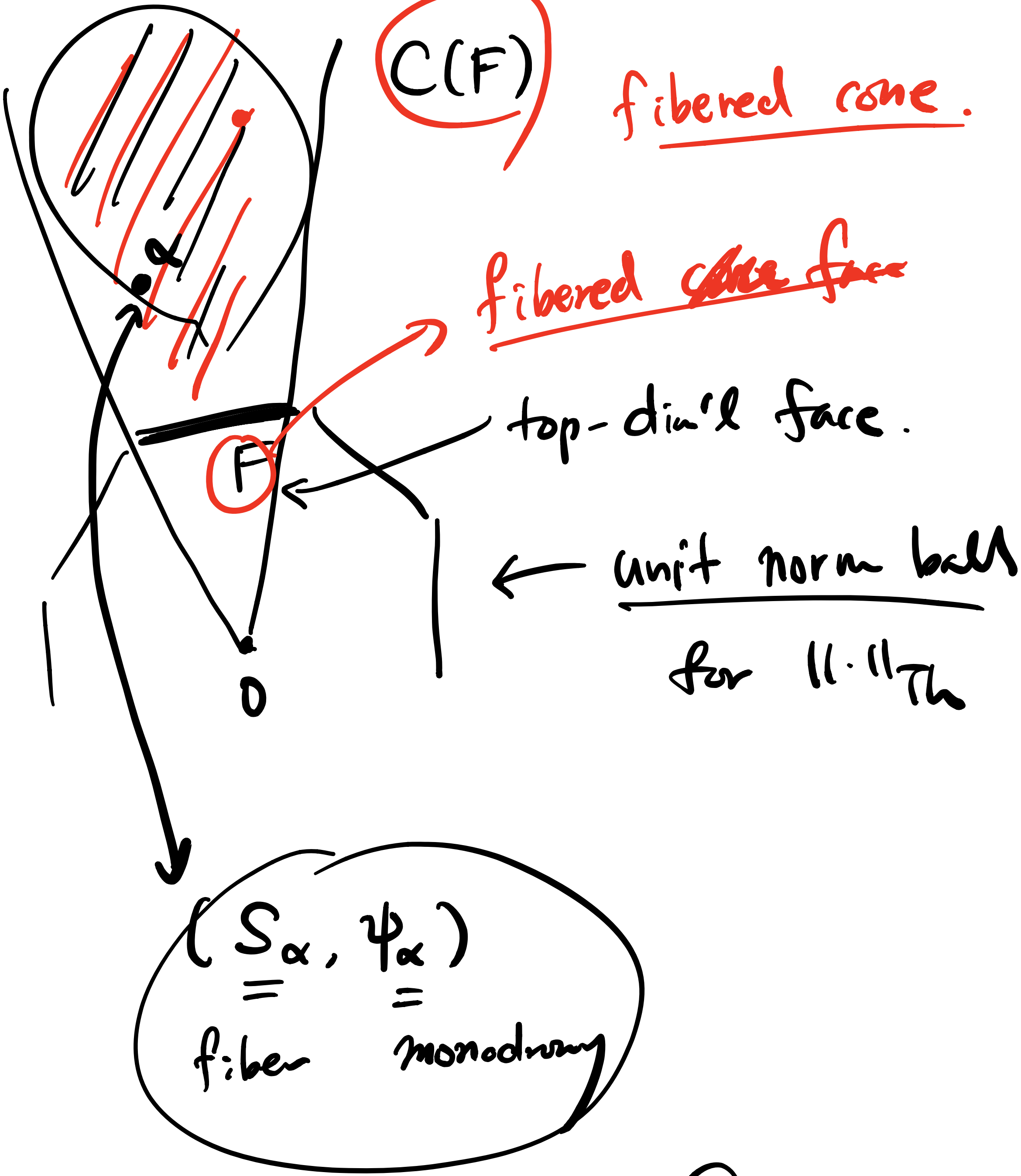
$$\|[S]\|_{Th} := \min_{S' \text{ rep of } [S]} \sum_{S_i \text{ components of } S'} \max\{0, -\chi(S_i)\}.$$

linearly extends to $H_2(M;\mathbb{R})$.

$\Downarrow$ M closed. hyperbolic 3-manifold

$\underline{\underline{M}}$ mapping torus





Thm (B.-Kin-Shin-Wu). (M) (F) $\exists\, C > 0$ s.t.

① $\forall\, \alpha \in C(F)$, $\ell_c(\psi_\alpha) \le \dfrac{C}{|\chi(S_\alpha)|}$

② For all but finitely many $\alpha \in C(F) \cap L$,

$\langle\langle \psi_\alpha \rangle\rangle = \mathrm{Mod}(S_\alpha)$.

1-dim'l rational

subspace.

Lem. Let $f \in \mathrm{Mod}(S_g)$, $g \geq 3$.

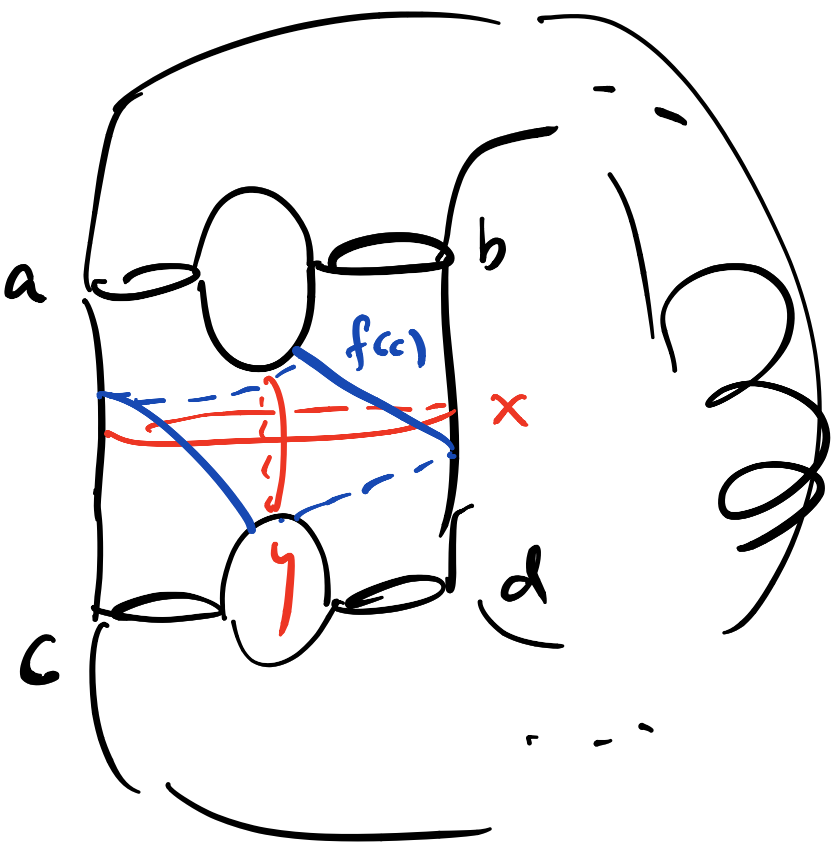
Suppose $\exists$ a non-separating s.c.c. c in S_g

s/t $i(c, f(c)) = 0$

& $[c] \neq [f(c)] \in H_1(S_g)$.

Then $\langle\langle f \rangle\rangle = \mathrm{Mod}(S_g)$.

"proof":



Latern relation

$$T_a T_b T_c T_d = T_{f(c)} T_x T_y.$$

$$T_d = T_c^{-1} T_{f(c)} \; T_a^{-1} T_b \; T_b^{-1} T_y.$$

$$T_c^{-1} (f T_c f^{-1})$$

$$= (T_c^{-1} f T_c) f^{-1} \in \langle\langle f \rangle\rangle$$

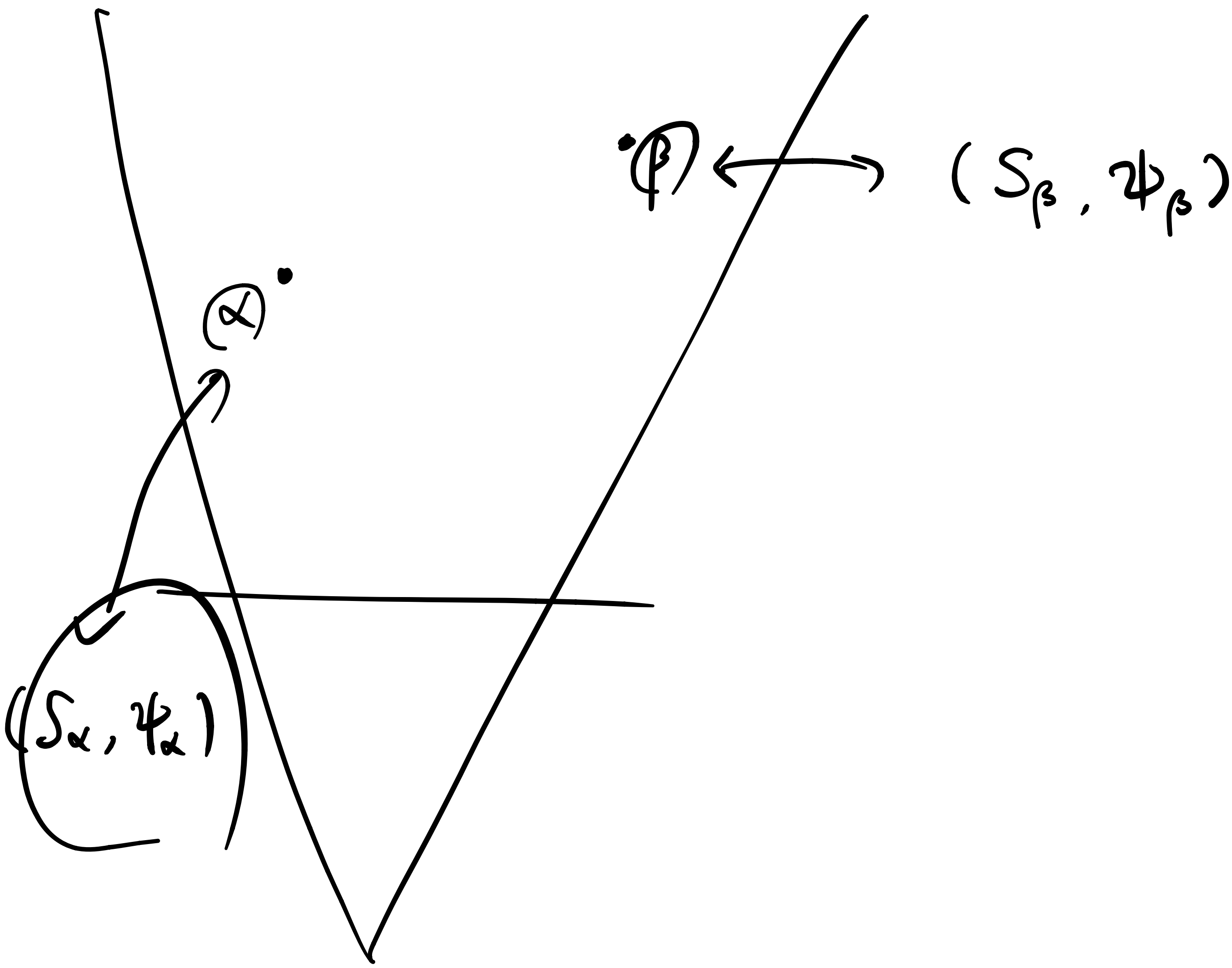
$c. \; f(c) \xrightarrow{h} a. \; b$

$\vdots$

$\longrightarrow T_a^{-1} T_b \in \langle\langle f \rangle\rangle$

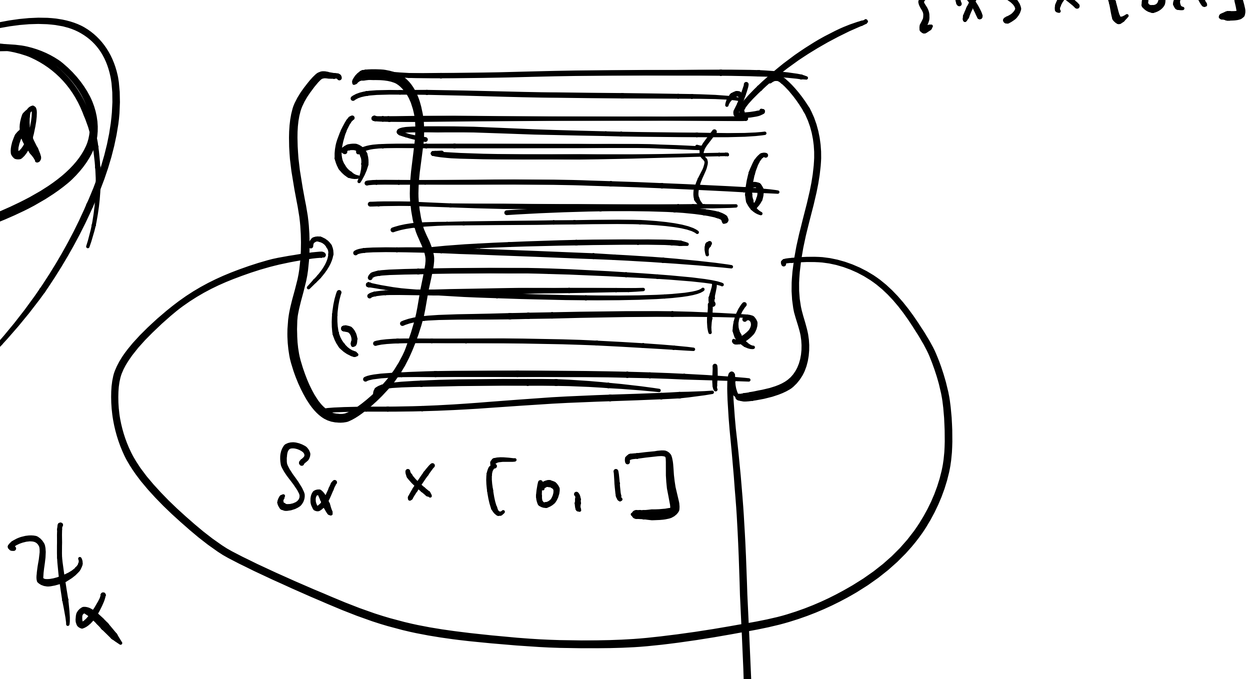
$T_d \in \langle\langle f \rangle\rangle$

"



$(\alpha + n\beta)_{n\in\mathbb{N}}$

Fried

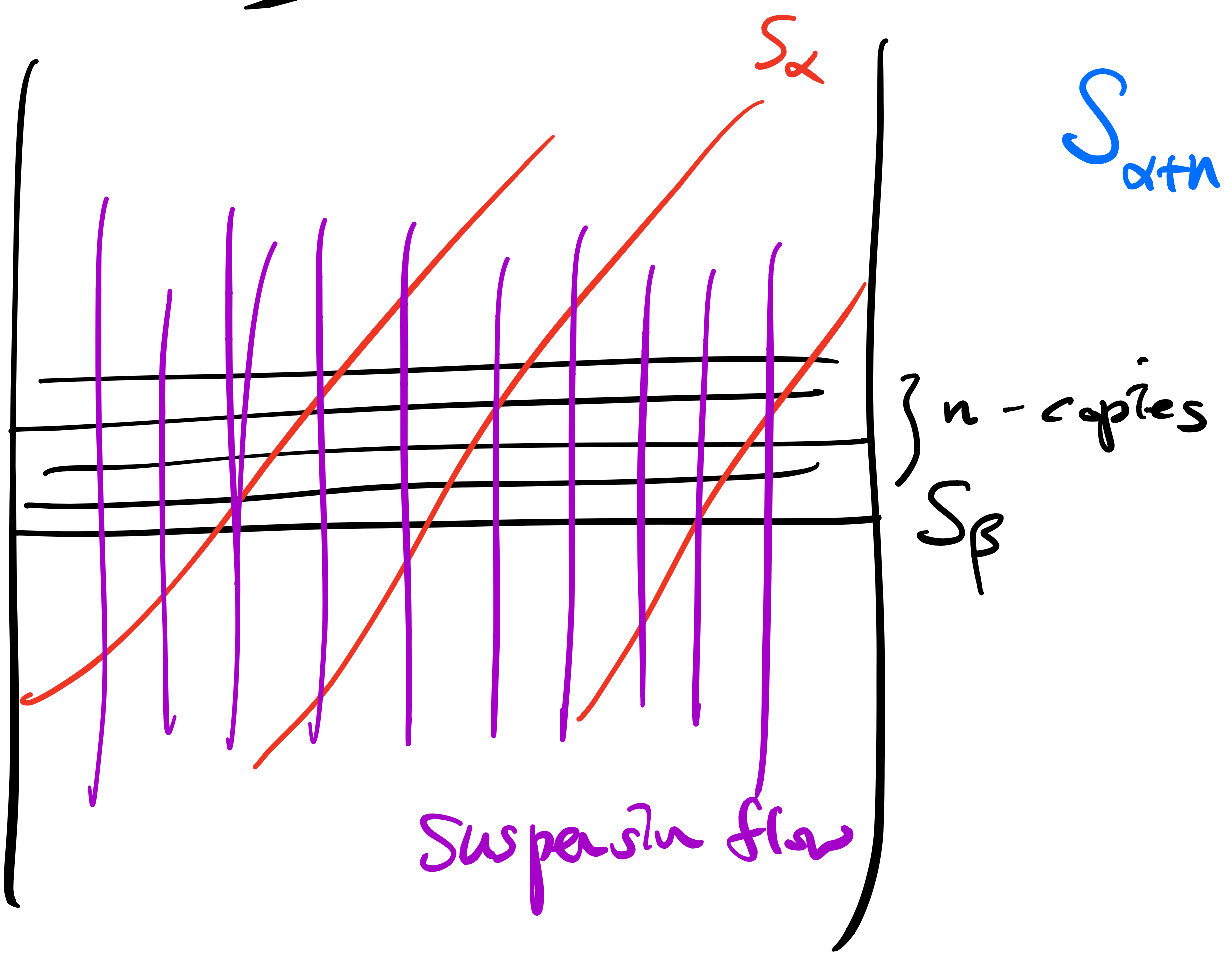


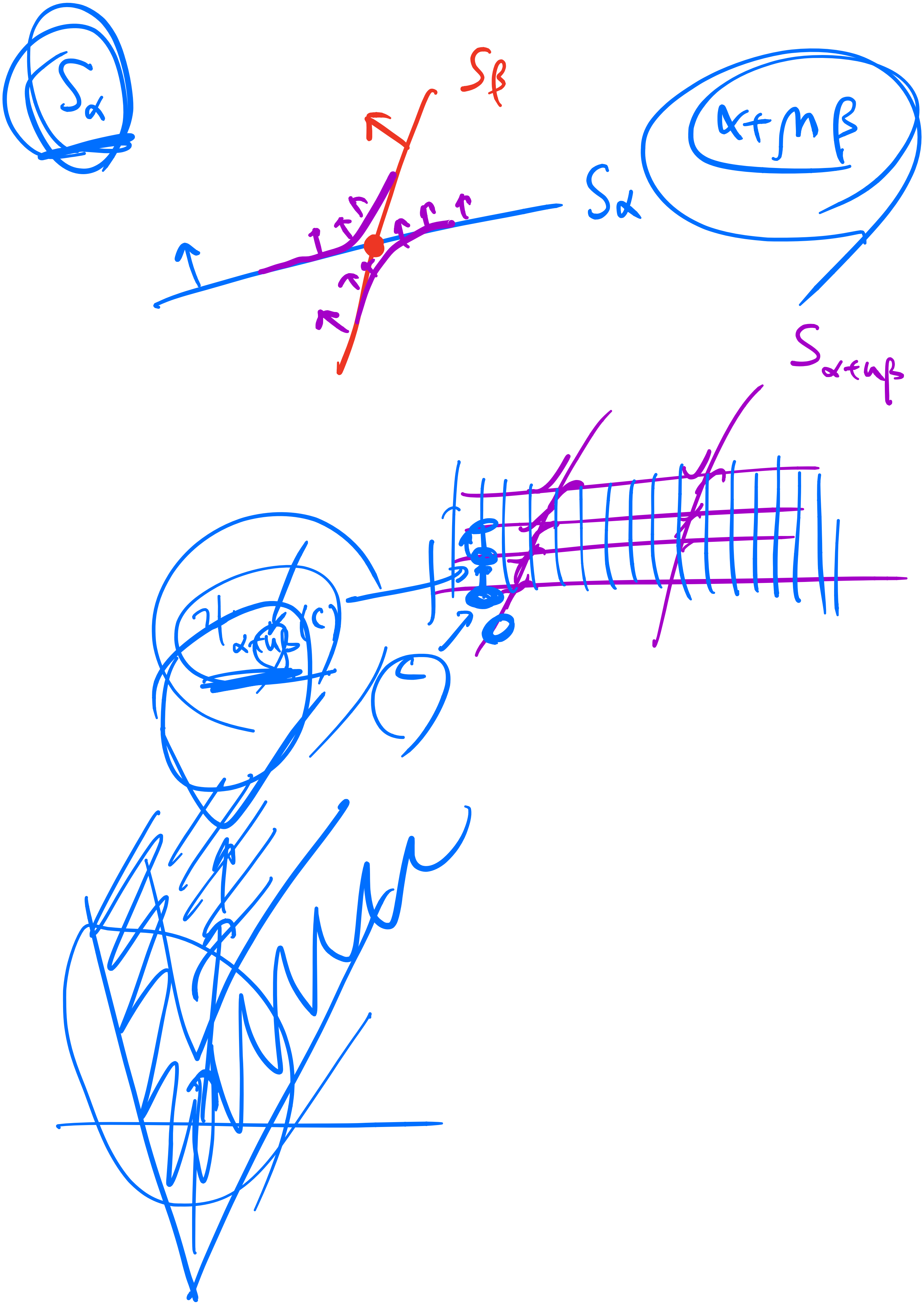
Suspension flow

α, β

$\alpha + n\beta$

$S_{\alpha+n\beta}$





S_α
S_β
S_α
$\alpha+n\beta$
$S_{\alpha+n\beta}$
$\mathcal{H}_{\alpha+n\beta}(C)$
C