$\Gamma_1, \ldots, \Gamma_n$ graphs. $\quad u(\phi) = (u_1, \ldots, u_n)$

$$V\left(\mathop{\ast}_{i=1}^{n} \Gamma_i\right) = \left\{ \left\{ \underbrace{v_1 \ldots v_\ell}_{=\tilde{v}} : v_k \in \Gamma_{i_k},\ i_{k+1} \neq i_k;\ v_{k+1} \neq u_{i_{k+1}}(v_1 \ldots v_k),\ k \in \{1, \ldots, \ell-1\} \right\} \right\}$$

$$= \bigcup_{\ell=0}^{\infty} V^{(\ell)}$$

$$u_i(\tilde{v}) = \begin{cases} v_\ell & \text{if } v_\ell \in \Gamma_i \\ u_i(v_1 \ldots v_{\ell-1}) & \text{if } v_\ell \in \Gamma_i \end{cases}$$

$$E\left(\mathop{\ast}_{i=1}^{n} \Gamma_i\right) = \left\{ \{\phi, v_1\} : \{v_1, u_i\} \in E(\Gamma_i) \right\}$$

$$\cup \left\{ \{\tilde{v}, \tilde{v}v_{\ell+1}\} : \{v_{\ell+1}, u(\tilde{v})\} \in E(\Gamma_j) \right\}$$

$$\cup \left\{ \{v_1 \ldots v_{\ell-1} v'_\ell, \tilde{v}\} : \{v_\ell, v'_\ell\} \in E(\Gamma_i) \right\}$$

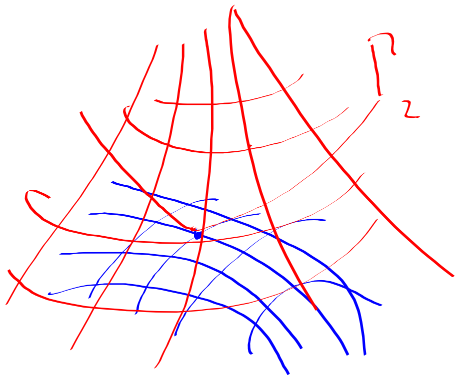
For $\tilde{v} = v_1 \dots v_e$, the $\Gamma_h$-sheet containing $\tilde{v}$ is

$$\begin{cases} \{ v_1 \dots v_{e-1} v_e' : v_e' \in \Gamma_i \} & \text{for } h = i \\ \{\tilde{v}\} \cup \{ \tilde{v}\, v_{e+1} : v_{e+1} \neq u_h(\tilde{v}) \} & \text{for } h \neq i \end{cases}$$

Spans a copy of $\Gamma_i$

spans a copy of $\Gamma_h$.

where $v_e \in \Gamma_i$.



For $\tilde{v} \in V(\overset{n}{\underset{i=1}{\ast}} \Gamma_i)$ there graph isomorphisms

$$\varphi_{(\tilde{v},i)} : \Gamma_i \longrightarrow \left(\overset{n}{\underset{i=1}{\ast}} \Gamma_i\right) \qquad \text{with } \varphi_{(\tilde{v},i)}(u_i(\tilde{v})) = \tilde{v}.$$

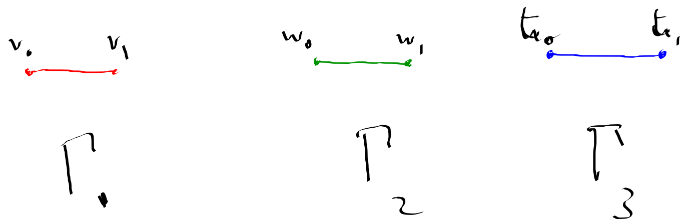
$\varphi_{(\tilde{v},i)}(\Gamma_i) = \Gamma_i$-sheet at $\tilde{v}$.

Each edge in $\overset{n}{\underset{i=1}{\ast}} \Gamma_i$ is contained in a unique $\varphi_{(\tilde{v},i)}(\Gamma_i)$

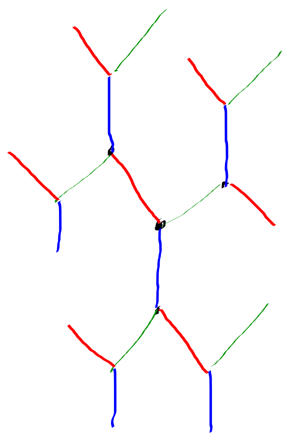
$V\left(\overset{n}{\underset{i=1}{\ast}} \Gamma_i\right)$ is partitioned by the $\Gamma_i$-sheets for each $i$.

$E(\overset{n}{\underset{i=1}{\ast}} \Gamma_i)$ is partitioned by the edge sets of the $\Gamma_i$-sheets, $i = \{1, .., n\}$.

## Example 1




$\underline{u}(\emptyset) = (v_0, w_0, t_0)$



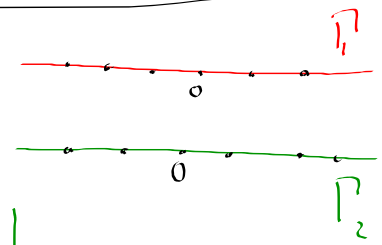
## Example 3

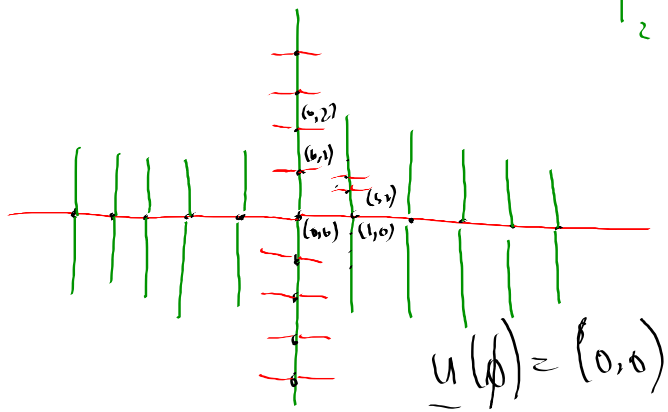
$\Gamma_1 = \bullet$ , $\Gamma_2 = \bullet\!-\!\!-\!\bullet$

$\Gamma_1 * \Gamma_2 = \bullet\!-\!\!-\!\bullet$

$\cong \Gamma_2$

## Example 2




$\underline{u}(\emptyset) = (0,0)$

Remark: $\Gamma_1 \times \dots \times \Gamma_n$ also has the properties that

(1) Each vertex is contained in an isomorphic copy of $\Gamma_i$ & these copies partition the vertex set.

$\Gamma_i$ = sheets $(v_1, v_2, \dots, \Gamma_i, \dots, v_n)$

(2) Each edge is contained in exactly one $\Gamma_i$-sheet.

$\{(v_1, \dots, v_n), (v_1', \dots, v_n')\}$ is an edge if there is $i$ s.t. $(v_i, v_i') \in E(\Gamma_i)$

& $v_j = v_j'$ if $j \neq i$.